Global instability of an inviscid compressible flow over a cavity

Fabien Méry and Grégoire Casalis

DMAE, ONERA, Toulouse, France

Abstract. This paper is devoted to the hydrodynamic stability computation of a compressible and high-Reynolds flow over a cylindrical cavity. The approach consists in determining amplified solutions of a partial derivative eigenvalue problem pertaining to linear instability of two-dimensional basic flow using finite elements. The results are compared to former results using classical method.

Keywords: Finite element method, Global linear stability, Kelvin Helmholtz modes

INTRODUCTION

Several flight tests showed the occurence of a tone generated by circular vent holes on the pressure side of a wing during approach conditions (Mach numbers ranging from 0.1 to 0.3)[1]. The french AEROCAV project aims to study the whistle, to understand the mechanisms enhanced in this geometry, and to propose a final approach to reduce this whistling by optimising the shape of these cavities. In order to understand the mechanism of such phenomenon, a linear stability study can be useful to identify the resonant modes and so to determine their role in the noise generation. This paper presents the results of the global linear stability of a flow over a cavity. The mean flow is computed by a Large Eddy Simulation method (LES): the mean flow is compressible and corresponds to a high-Reynolds number case. As the mean flow is fully non-parallel, the classical local stability theory is not appropriate. We must use the method initiated by Tatsumi [2] and developed by Theofilis: the so-called bi-global stability theory [3]. Recently, stability studies of laminar flows have been driven on the rectangular cavity [4, 5] ; these studies were using a low Reynolds number configurations with full compressible Navier-Stokes equations. Such fundamental studies represent a starting point for the understanding of the cavity flow phenomena but cannot be used in the present configuration. This paper gives an applied point of view on the cavity flow issues, and aims to deal with practical problems.

DESCRIPTION OF THE CONFIGURATION AND MATHEMATICAL FORMULATION

Geometry and Mean Flow

The configuration retained for this study is related to the french AEROCAV project. The cavity has a cylindric shape, with a diameter D equal to $0.1\,m$, the depth W is also equal to $0.1\,m$. The inflow freestream velocity is $U_\infty = 70\,m.s^{-1}$, corresponding to an inflow Mach number of $M = 0.2$. The cavity diameter based Reynolds number is about 450 000. The mean flow is obtained by the time-average field of a LES performed by Mincu [6] with the ONERA code "Flu3M". This simulation is fully three-dimensionnal and turbulent ; it has about 13 million mesh points with 40 points in the boundary layer. To obtain dimensionless quantities (velocity components and time), we use the cavity depth W, the freestream velocity U_∞ and finally the ratio W/U_∞. The stability studies were conducted considering the Euler equations in the compressible form for the streamwise median plan through the cavity (see fig 2).

Perturbation equations

In the median plan, the mean flow is two-dimensionnal in the sense that the two velocity components $(\bar{u}(x,y), \bar{v}(x,y))$ are functions of x and y only. We assume that the mean density and the mean pressure

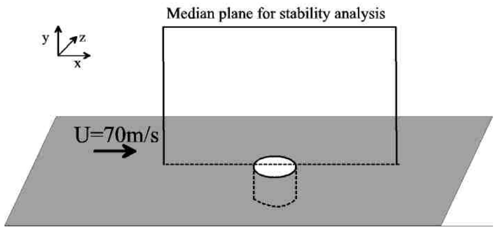

FIGURE 1. Schema of the studied configuration

are nearly constant, which is confirmed by the LES results. According to the theory of linear stability, a systematic small perturbation is superposed to the mean flow. The perturbations are computed using an ideal gas with a constant specific heat. Thanks to the nearly constant mean density and mean pressure, the entropy conservation equation and the equation of mass continuity can be grouped. We consider the perturbations in the form of normal modes: $\widetilde{q}(x,y,t) = \hat{q}(x,y)exp(-i\omega t)$ with $\hat{q}(x,y)$ and ω complex numbers. This leads to an eigenvalue problem via a partial derivative equation system. The linearised inviscid compressible Euler equations system becomes :

$$\left\{ \begin{array}{rcl} M^2(-i\omega\hat{p} + \bar{u}\hat{p}_{,x} + \bar{v}\hat{p}_{,y}) + (\hat{u}_{,x} + \hat{v}_{,y}) & = & 0 \\ (-i\omega\hat{u} + \bar{u}_{,x}\hat{u} + \bar{u}_{,y}\hat{v} + \bar{u}\hat{u}_{,x} + \bar{v}\hat{u}_{,y}) + \hat{p}_{,x} & = & 0 \\ (-i\omega\hat{v} + \bar{v}_{,x}\hat{u} + \bar{v}_{,y}\hat{v} + \bar{u}\hat{v}_{,x} + \bar{v}\hat{v}_{,y}) + \hat{p}_{,y} & = & 0 \end{array} \right. \tag{1}$$

with the notation $f_{,z} = \frac{\partial f}{\partial z}$. There are three unknowns : $\hat{u}$, $\hat{v}$, $\hat{p}$, the amplification functions for the streamwise and vertical fluctuating velocity components and for the fluctuating pressure.

On rigid boundaries, the normal velocity perturbation must vanish, this leads to physical Dirichlet conditions. The other boundary conditions will be specified further on.

NUMERICAL METHODS

Finite element method

Usually, stability problems are solved using spectral methods in order to have a good accuracy. However, in the cavity case, these spectral methods will need a multi-domain formulation and the geometry should be defined once for all. Conversely, the finite element method is a good choice for solving partial differential equations over complex geometries even if the accuracy will be not so good than the one using a spectral method. We use the FREEFEM++ software (see http://www.freefem.org) to perform these finite element computations. The previous system (1) is written in variationnal form and we are using a finite element space composed of P1 piecewise linear continuous finite elements plus bubble. This element type gives an increased degree of freedom which corresponds to the barycentric coordinates. After superposition of element matrices, the temporal mode stability (1) is expressed as a generalized eigenvalue problem in the form of

$$\mathbf{A}q = \omega\mathbf{B}q \tag{2}$$

where $\mathbf{A}$ and $\mathbf{B}$ are matrices and $q = \{\hat{p}, \hat{u}, \hat{v}\}$ is the eigenfunction vector.

Arnoldi methods, Convergence

Computing the whole eigenmode spectrum takes too much time. The Arnoldi's method [7] seems to be appropriate in the present case. Actually, we are using a shift value and the algorithm tries to converge on eigenvalues close to the shift value. The FREEFEM++ software can also perform the Arnoldi method thanks to UMFPACK [8] (which is a sparse matrix solver) and ARPACK coupling. It is possible to build a "pseudo-spectrum" by varying the shift values. These computations are carried out for different mesh grids and different shift values in order to identify physical modes. Regarding the mean flow, the critical phenomenon is the shear layer over the cavity where the strong velocity gradient triggers Kelvin-Helmholtz instabilities [9]. These unstable modes are difficult to compute as they are concentrated in the shear layer zone. Consequently, the mesh is tight up to an average of 35 vertices in this zone. Furthermore, we impose the fluctuating velocity components to vanish on the non-rigid boundaries.

The difficulty of this part is to analyse each possible converged mode. So each possible eigenfunction must be analysed. We are actually interested in unstable modes, i.e. modes with positive ω_i values. The corresponding modes must have the characteristics which could be linked to physical phenomena. These constraints allow us to identify "physical modes", which are analysed in the following.

NUMERICAL RESULTS

Kelvin-Helmholtz modes

Figures 2 and 3 show four modes ($\omega_r = 3.22$, $\omega_r = 5.83$, $\omega_r = 8.01$, $\omega_r = 9.77$) which seem to be quasi-independant on different mesh refinements. These modes are localised in the shear layer zone. Everywhere else, the fluctuating velocity components are null. Therefore, these modes are the expected Kelvin-Helmholtz modes. We define the Strouhal number as

$$St = \frac{\omega_r}{2\pi}$$

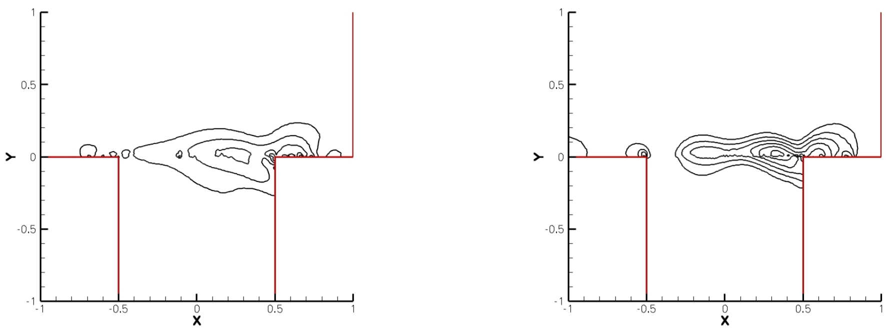

FIGURE 2. Unstable Kelvin-Helmholtz modes I (left) and II (right). Iso-value of the pressure modulus perturbation for mode I ($St_I = 0.51$) and mode II ($St_I = 0.92$)

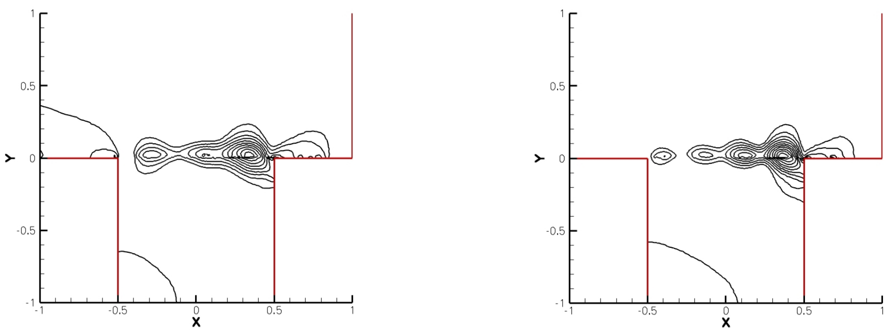

FIGURE 3. Unstable Kelvin-Helmholtz modes III (left) and IV (right). Iso-value of the pressure modulus perturbation for mode III ($St_{III} = 1.28$) and mode IV ($St_{IV} = 1.55$)

The semi-empirical Rossiter's formula predicts dominant harmonics [10]. Indeed, flow perturbations by impacting on downstream edge excite the shear layer and a particular tuning between the flow perturbations roundtrip and the number of vortices in the shear layer may introduce self-sustained oscillations. In our case, we will use a Rossiter formula which is efficient for low Mach numbers, it has been established by Block [11]:

$$St_{Rossiter_n} = \frac{n}{M(1 + \frac{0.514}{D/W}) + \dfrac{1}{\kappa(D/W)}} \tag{3}$$

TABLE 1. Comparison between Strouhal numbers predicted by formula 3 and the Strouhal numbers of unstable modes

	MODE I	MODE II	MODE III	MODE IV
Strouhal number of global unstable mode	0.51	0.92	1.28	1.55
Strouhal number of Rossiter mode	0.51	1.02	1.53	2.04

with $\kappa(D/w)$ an empirical parameter. This relationship takes into account the shear layer thickness. In table 1, we can notice that global unstable modes I and II are almost the same as those predicted by Rossiter. And mode IV corresponds to the mode III of Rossiter. These surprising results are independent on the acoustic phenomena which tend to give another justification of the Rossiter formula. Indeed, the Rossiter self-sustained oscillations seem to be conditioned by the unstable Kelvin-Helmholtz modes.

A second aspect of the study concerns the eigenfunctions themselves. Rowley and Colonius [12] have performed several Direct Numerical Simulations for a compressible flow over a rectangular cavity. The cavity aspect ratio was $D/W = 2$ (1 in the present study). They performed computations for a Mach Number $M = 0.6$ (0.2 in the present study). They demonstrated that the two first Rossiter's modes are amplified. They have computed discrete Fourier transform for Rossiter's second mode [10]. In the full paper version, we will show that eigenfunctions of mode II are in good agreement with the results obtained in [12].

CONCLUSION

This study is a first analysis of global instabilities of a high Reynolds number and compressible flow over a cylindrical cavity using linearized Euler equations. This paper shows the efficiency of a simple method (global instability theory) on complex flow configurations (high Reynolds number, complex geometry...). The results enable a new interpretation of the Rossiter formula, particularly, for modes I and II.

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