Automatic Transition Predictions using Simplified Methods

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Laminar-turbulent transition remains a critical issue in a number of cases, among which drag reduction, performance prediction of high lift systems, improved accuracy in general CFD, or reduction of computation cycles for development of optimization tools. Transition delay remains one of the most promising technologies for reducing air transport energy consumption, through natural or hybrid laminar flow control. The use of linear stability theory, either local or non local, remains rather demanding in term of knowledge, and user interaction. Hence a demand for simplified, robust and accurate transition prediction tools to be inserted into general flow solvers, of boundary layer or RANS types. The problem can be solved by developing transition criteria or database methods. In this last case, characteristics of an actual flow are derived from known solutions of model flows. ONERA has long been involved in the development of such methods, and the present paper aims at providing a comprehensive view of the tools developed in the second category, applicable from low speed 2D to transonic 3D flows, and even to 3D supersonic flows.

Nomenclature

- \( U_e, W_e \): external velocity components
- \( \nu = \frac{\mu}{\rho} \): kinematic viscosity
- \( \alpha, \beta \): wavenumbers (complex). Their real parts are the components on the wavevector
- \( \sigma = -\alpha_i \): amplification rate
- \( \delta_1 = \int \left( 1 - \frac{U(y)}{U_e} \right) dy \): displacement thickness
- \( \theta = \int \frac{U(y)}{U_e} \left( 1 - \frac{U(y)}{U_e} \right) dy \): momentum thickness
- \( \omega = \frac{2\pi \delta_1}{U_e} \): reduced pulsation
- \( F = \frac{2\pi \sigma^* \mu_e}{\rho_e U_e^2} = \frac{\omega_r}{R \delta_1} \): reduced frequency

Subscripts:
- \( e \): at the boundary layer edge

1. Introduction

Automatic and robust laminar-turbulent transition prediction tools are still in demand, for improving accuracy of flow computation or development of optimization tools. A number of models have been developed at ONERA, and have

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been combined into a fairly complete prediction tool which may be inserted into boundary layers or Reynolds Averaged Navier Stokes (RANS) codes. The aim of the present paper is to provide a comprehensive view of these models, including examples of applications and future extensions. Traditional approach for transition prediction is based on the linear stability theory (LST), either local or non local. The addition of second order terms, curvature and non parallelism, in the non local approach does not significantly improve correlation of computation to experimental results. So, local theory remains widely used for practical applications. While stability analysis describes how small, pre-existing perturbations will grow in the boundary layer through a normal mode response, the εN method correlates an amplitude level with the beginning of turbulence. Here, NC = log (A/Ao), where A/Ao is the amplitude ratio between the current location and a reference, upstream one. The two most common strategies, envelope and NC/NTS, will be discussed later. Stability calculations are rather demanding in term of user interaction (no fully automatic code yet), and require a precise description of the boundary layer, imposing conditions on the mesh definition (at least about 40 points in the boundary layer) and on the numerical scheme (low dissipation) in case of a RANS approach. Two separate questions arise, first the development of simplified stability methods, and second their coupling to RANS codes. Concerning the first point, a number of contributions have been made, see ref. 3 to 12, which generally fall into the database approach, where solution for an actual boundary layer profile is obtained from the knowledge of the pre-computed stability solutions for a family of profiles, typically of a self-similar family like the Falkner-Skan one in 2D. The procedure then intends to predict the stability characteristics of actual (non-similar) profiles from those of the model profiles. Various procedures and models have been developed, which fall into 3 categories, look-up tables, analytical models and neural networks. In the two first cases, the stability characteristics of a given profile will be determined using an interpolation table or some analytical functions of selected parameters, in the last case a neural network will provide the desired parameters after a specific training of the network. The range explored by the family of similar profiles may vary depending on the applications. Due to his interest with interactive boundary layers, Drela introduced in his database 2D profiles with separation. Other contributors concentrated on attached flows, in relation with direct mode boundary layer solvers considering configuration without separation. In case of ONERA, the self similar Falkner-Skan family was considered for the development of Tollmien-Schlichting, or TS, instabilities, while models for cross flow, or CF, modes were based on typical profiles obtained downstream of the attachment line for a series of low speed and transonic realistic swept wing flow cases. A similar model was developed for TS instabilities by Stock. At ONERA, three separate models were developed, respectively dealing with longitudinal, or TS instabilities, then traveling crossflow, or CF instabilities, and finally with stationary crossflow modes, which we call CF0. Concerning the coupling of stability based methods to RANS codes, several approaches have been explored. ONERA introduced into the elsA code transition criteria for TS and CF instabilities. These criteria were developed from results of linear stability and transition correlations. Examples were presented in ref. 14 in the context of high lift flows. DLR, INTA and others have developed, for wing surfaces, an automatic coupling to a boundary layer code inside which database methods are used for transition prediction. A presentation of European activities in this field was recently published, see ref. 16. As there is no example of automatic, internal coupling of a RANS code with a stability approach capable of using directly the RANS velocity profiles, there is still room for much progress.

II. Linear stability and the solution form

Consider the swept wing depicted in figure 1, with a \( \Psi \) sweep angle. A local wing coordinate system is defined taking \( x \) as the normal to the leading edge, \( y \) normal to the wall and \( z \) in spanwise direction. \( U, W \) are the velocity components of the mean flow in \( x \) and \( z \) directions. \( \theta \) is the angle between \( x \) and \( x_c \), with \( x_c \) the tangent to the external streamline at a point.

Figure 1: Wing geometry and coordinates

Figure 2: Geometrical definitions
In the frame of local stability theory, $\frac{\partial G}{\partial x} = \frac{\partial G}{\partial z} = 0$, where $G = U, W$. The linear growth of small perturbations, added to the base flow is considered using stability theory. Solutions are written in the form

$$g(x,t) = g(y) e^{i(\alpha x + \beta y - \omega t)}$$

where $g$ is any fluctuating quantity, $\alpha = \alpha^* \delta_1$, $\beta = \beta^* \delta_1$. In spatial theory, $\omega = F/R\delta_1$ is real and both wavenumbers $\alpha, \beta$ are complex. The reduced frequency is defined as $F = \frac{2\pi \nu}{U_c^2}$.

Real parts of the wavenumbers define the wavevector $k$, see fig. 2, at angle $\phi$ from the $x$ direction, and $\psi$ from the local velocity direction:

$$\phi = \tan^{-1}\left(\frac{\beta}{\alpha}\right), \quad \psi = \psi + \theta.$$ Given these definitions, the local stability equations (LST) for a 3D incompressible flow is the well know Orr-Sommerfeld equation, of fourth order, while for compressible flow a sixth order system is obtained. The incompressible equation is

$$v''' = 2(\alpha^2 + \beta^2)v'' + (\alpha^2 + \beta^2)v' + R - i(\alpha U + \beta W - \omega)v = 0$$

with homogeneous boundary conditions on $v$ and $v'$. Here, the primes denote derivatives with respect to the $y$ coordinate, and $R$ is a Reynolds number. Solutions only exist for eigenvalues ($\alpha, \beta, \omega$). In practice, ($\beta, \omega$) – or ($\psi, \omega$) - are imposed, and $\alpha$ is obtained as an eigenvalue of the system. The growth rate of solutions is then given by the imaginary part $-\alpha_i = -\sigma$, and the N factor is obtained by integrating this growth rate.

As there are more unknowns than equations, parameters need to be imposed before solving the equations. Several methods exist, referred to as integration strategies. A complete discussion - see ref. 17 and 18 - is outside the scope of this work. In the present context, two approaches are considered, the envelope method and the two N-factor methods NTS/NCF, with the following definitions:

The envelope N factor is obtained by integrating the largest amplification rate, $N_{env} = \max_f \int \sigma_{\psi} dx$. The inner maximization may be defined over $\psi, \beta^*$ or $\lambda$. The longitudinal N-factor is defined by integrating $\sigma_{\psi=0}$, considering waves with wavevectors parallel to the outside velocity: $N_{TS} = \max_f \int (\sigma_{\psi=0}) dx$. This definition may be extended to consider longitudinal oblique waves, with $N_{env} = \max_f \int \sigma_{\psi < \psi_{\text{max}}} dx$, where $\psi_{\text{max}} = 60$ to 80 deg. The envelope of envelope crossflow N-factor is obtained by imposing a constant wavelength or a constant spanwise wavenumber. $N_{CF} = \max_f \int (\sigma|_{\beta^* = \epsilon}) dx$. $N_{CF}$ may be strictly based on stationary modes ($\psi=0$), suppressing the outer maximization, or in a concurrent definition may include traveling waves ($\psi > 0$). NTS and NCF may also be computed based on incompressible stability equations. Whatever the definitions used, each method needs to be calibrated using some experimental database.

As the envelope method was the most commonly used at ONERA, the first database developments were restricted to this approach. An extension to stationary modes and a NTS/NCF method were later added with slightly different definitions for NCF, as follows: $N_{CF0} = \int \max_{\beta^* \in CF} (\sigma|_{\psi=0}) dx$ and $N_{CF} = \max_f \int \max_{\beta^* \in CF} (\sigma) dx$.

Figure 3: Comparing NTS and NCF0 for LST and database, for a low speed case

Figure 4: Consequences of various definitions for NCF
The database $N_{TS}$ can be considered equivalent to the exact stability constant $\psi = 0$ N-factor, and the stationary database $N_{CF}$ can be considered equivalent to the zero frequency envelope calculation (since only crossflow modes exist at zero frequency), as can be seen on figure 3. (Figures 3 and 4 correspond to the test case presented on figure 12). On the other hand, figure 4 illustrates the different ways to estimate $N_{CF}$ and $N_{CF0}$ for the same low speed case. Very different results are obtained depending on the definition, hence for transition prediction calibration must be determined for each formulation.

III. Database approach for longitudinal instabilities

Longitudinal, or TS instabilities, are governed by viscosity. A first model of such instabilities for 2D low speed flow was proposed by Vialle & Arnal\(^4\) in 1984 based on a set of exact stability solutions of attached Falkner-Skan self similar profiles in 2D flow. The incompressible shape factor $Hi = \frac{\delta_1}{\delta}$ was used as the key parameter.

The simplified stability approach, or ‘database method’, provides an estimation of the growth rate $\sigma$ directly from mean flow parameters and the boundary layer profile characteristics. Starting idea is that the Reynolds number variation of growth rates obtained solving the exact Orr-Sommerfeld equations can be represented, for a given profile, using two half parabolas as shown on figure 5. Extension to decelerated flow ($Hi > 2.59$) can be obtained using an added ‘inviscid’ parabola, as shown on figure 6. The effect of compressibility was introduced into the model, using the external Mach number as an additional parameter\(^3\).

For a given mean flow $(Hi, Me)$ and frequency $F$, the amplification curve is approximated as:

$$
\sigma = \max[\sigma_f, \sigma_i]
$$

$$
\sigma_f = \sigma_{Hi}, f \left(1 - \frac{R_{0,f} - R_{M,f}}{R_{f} - R_{M,f}}\right), R_{0,f} = R_{f} \text{ if } R_{0} < R_{M,f}, \text{ and } R_{0,f} > R_{M,f}, \text{ if } R_{0} > R_{M,f}
$$

$$
\sigma_{Hi} = B_f \left(1 - \frac{F}{\bar{F}_{Hi}}\right), \sigma_{if} = \min\left\{B_f \left(1 - \frac{F}{\bar{F}_{if}}\right), \sigma_{Hi}\right\}
$$

With

$$
R_{M,f} = K_{1f} F_{1f}\frac{E_{1f}}{1}
$$

$$
R_{0,f} - R_{M} = R_{M} C (F_c - F), \quad R_{0,f} = R_{0f}
$$

$$
R_{V,f} = K_{2f} F_{2f}\frac{E_{2f}}{1}
$$

Using these definitions, 15 parameters need to be determined as functions of $Hi$ and $Me$:

$(B, F), (E_1), (K_1, E_1), (C, F_c), (K_2, E_2), (\sigma_{M,Me})$

This is realized using a two-entry look-up table. Amplification rates are then obtained as a function of the Reynolds number $R_{0f}$ based on the incompressible (resp. compressible) displacement thickness for incompressible (resp. compressible) flow.

Tables are defined for $2.22 < Hi < 4.023$, and $0 < Me < 1.3$.

Use is limited to profiles without backflow. In order to calculate stabilizing regions, where N-factors decreases, extension to stable amplification rates has been added by extrapolating the parabolas with a line tangent at the zero crossing point, as shown on figure 6.

There are two ways for using this model: First, as originally conceived, it can be applied to the velocity component profile in the direction of the external streamline. This defines a TS amplification rate, which can be integrated into N-factor curves, producing the $N_{TS}$ N-factors after a

![Figure 5: The basic parabola model](image)

![Figure 6: Extended parabola model for TS waves](image)
frequency envelope. For transonic flows, the amplification rates generated by the method correspond to the most unstable oblique wave in exact calculation.

Second, velocity profiles projected in a $\Psi$ direction can be considered, using Gaster’s relation and the Stuart theorem in the way described in the next section. This allows optimization in $\Psi$ direction, and can be used to define an envelope N-factor (still limited to viscous instabilities). This second method slightly improves the longitudinal N-factor computation, and generates a $\Psi$ dependence. It may also complement the computation of the envelope by including highly oblique longitudinal waves (see fig. 7 and corresponding discussion).

IV. Simplified model for inflectional instabilities

Cross-flow instabilities are inflectional in nature, they are determined by the location and characteristics of the velocity profile inflection point. Considering 2D profiles with reverse flow near the wall, it was first shown in 1986 that 2 parameters had to be used, related to the characteristics of the inflection point. These are $U_j = u(y_j)$ and $P_j = y_j \left( \frac{\partial u}{\partial y} \right)_{y_j}$, where $y_j$ is the location of the highest inflection point.

This first model was developed for 2D separated boundary layers of the Falkner-Skan family, but was found inadequate when looking at inflectional profiles like those causing crossflow instabilities in 3D boundary layers. A new model was then created during the ELFIN II project by Casalis & Arnal and successfully applied to propagating crossflow instabilities.

In order to extend a 2D model to 3D mean flow, use of Stuart’s theorem and Gaster’s relation are necessary. In temporal theory, and for incompressible flow, Stuart’s theorem states that the growth rate $\sigma_{\phi}$ in any direction $\phi$ can be determined from the stability of the 2D velocity profile resulting from the projection of the original 3D velocity in that same $\phi$ direction. This projected 2D profile is defined as $U_\phi = \frac{\alpha}{k} U + \frac{\beta}{k} W$, where $k = \sqrt{\alpha^2 + \beta^2}$ is the modulus of the wavevector $k$. This applies in case of temporal theory, while boundary layer stability usually uses spatial theory. In what follows, $(x,z)$ defines some in plane local co-ordinate system. $\psi$ is the wave vector direction relative to the external streamline, with $\phi = \psi + \theta$. $\varphi_g$ is the group velocity direction with respect to the local $x$ direction. In most cases, $\varphi_g$ remains very close to $\theta$, within a few degrees, i.e. the group velocity direction remains very close to that of the external velocity. This property is used here, as well as in many instances of exact LST resolutions. Gaster’s relation gives a relation expressing spatial growth rate in some $\phi$ direction in term of temporal amplification $\omega_j$, group velocity modulus $V_g$ and direction $\varphi_g$:

$$-\sigma_{\phi} = \frac{\omega_j}{V_g \cos(\phi - \varphi_g)}$$

In $x$ direction, $\phi = 0$ and $-\sigma_x = \frac{\omega_j}{V_g \cos(\varphi_g)}$.

$-\sigma_x$ may be expressed in term of $-\sigma_{\phi}$ computed in some $\phi$ direction:

$$\sigma_x = \sigma_{\phi} \frac{\cos(\psi + \theta - \varphi_g)}{\cos(\varphi_g)}$$

and, noting $\varphi_g \approx \theta$,

$$\sigma_x \approx \sigma_{\phi} \frac{\cos(\psi)}{\cos(\theta)}$$

This relation gives the amplification in $x$ direction, used for N-factor integration, as a function of the amplification in the $\phi$ direction, computed using the velocity profile projected in that direction.

In case of an inflectional profile, the amplification rate is defined as before

$$\sigma = \sigma_M \left( 1 - \left[ \frac{R_{\delta_i} - R_M}{R_j - R_M} \right]^2 \right) \quad R_j = \begin{cases} R_0 & \text{if } R_{\delta_i} < R_M \\ R_j & \text{if } R_{\delta_i} > R_M \end{cases}$$

now with the following definitions:
\[
\sigma_M = aF + b\sqrt{F} + \sigma_\infty
\]
\[
R_M = k_M F^{-m_M}
\]
\[
R_0 = k_0 F^{-m_0}
\]
\[
R_1 = k_1 F^{-m_1}
\]

Where \(a, b, \sigma_\infty\) are analytical functions of \(U_i, P_i\), and the \((k, m)\) coefficients are functions of \(\sigma_\infty\).

Extension to compressible flow has been achieved by using the generalized inflection point, \(\frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) = 0\), and introducing the density value into \(P_i : P_i = \rho(y_i) y_i \left( \frac{\partial u}{\partial y} \right) \).

The final model applies only to traveling cross-flow instabilities \((F > 0)\), and in a range \(\psi < \psi_{\text{max}} < 90^\circ\). In practice, \(\psi_{\text{max}}\) is set between 88.5 and 89. deg. For non-zero frequencies, the two models for longitudinal and crossflow instabilities were later combined. The resulting database method produces, with high efficiency, an estimation of the stability characteristics of 3D boundary layers. Figure 7 compares the TS and CF growth rates to exact LST solutions, for a swept wing low speed case at a given, non-zero frequency. The method of profile projection is used here for both TS and CF. It can be seen on figure 7 that database TS growth rates remain close to exact growth rates up to \(\psi = 70^\circ\), and misses completely the CF contribution for larger \(\psi\). On the other hand, the CF model produces, around \(\psi = 80^\circ\), a good estimation of the most amplified CF instability.

In general, the TS models produce a quite good approximation of \(\sigma(\psi)\) over a wide \(\psi\) range, while the CF model only produces a good estimation of \(\sigma_{\text{CF, max}}\).

VI. Stationary modes

The previously described model is strictly limited to traveling instabilities, and indeed its precision increases with frequency. It is well suited for transition predictions on transonic swept wings, but does not allow comparisons to results obtained using two N-factor methods.

With the objectives of extending the previous model, and to allow such comparison, stationary modes were later considered.

This model was created based on exact stationary stability solutions of a set of low speed and transonic wings flows, with validation using a quite extensive set of cases. This part was realized by Perraud & Donelli\(^{10}\) in the ALTTA European project. In this case, the maximum growth rate is generally located at the junction of branches B1 and B2, as shown on figure 8 for exact solutions for several stations of a low speed case. In this case, the Reynolds variations of projected amplification rates should not be represented by parabolas, but by hyperbolas as visible on figure 9. Plotting the variations of \(\alpha_i R_\phi\) versus \(R_\phi\) produces almost straight lines, hence amplifications can be estimated as \(\alpha_i R_\phi = A R_\phi + B\).

Looking at figure 9, this expression may be written for branch one as \(\alpha_{i\phi} = \sigma_\infty \left( 1 - R_c / R_\phi \right)\), where \(\sigma_\infty\) represents the asymptotic value of the growth rate when the Reynolds number \(R_\phi\) tends to infinity, and \(R_c\) is a critical projected Reynolds number. These two parameters are equivalent to the coefficients \(A\) and \(B\) in the previous equation. Parametric variations show that \(R_c\) essentially depends on the parameter \(P_i\), while \(\sigma_\infty\) is a function of \(U_i\).
The two branches need to be modeled independently. In both case, \( \sigma_{\infty} \) is represented using a polynomial expression function only of \( U_i \), of third order for branch one and second order for branch two. An exponential function of \( P_i \) is used for \( R_e \) for branch one, and again a second order polynomial in \( R_e \) for branch two.

An additional condition, based on \( P_i \), allows to determine an upper \( \psi \) limit. A spline function is used to represent the junction between the two branches, because using the crossing point between the two branches would strongly overestimate the growth rate. The local extremum gives the maximum growth rate and the corresponding \( \psi \). This produces the largest stationary growth rate at any given station, so the \( N \) CF is an envelope over all values of the crossflow wavenumber \( \beta^* \).

Figure 10 shows a comparison of the exact and modeled amplification rate for a typical transonic case, for three different stations. Actual amplification rates (for the 3D profile) are shown, for which a very good agreement is obtained.

VII. Methods for transition prediction

The three models presented for TS, CF and CF0 have the advantage of being extremely fast to compute, and of being independent of some initial guess, a common difficulties of shooting methods. The TS growth rates only depend on integral boundary layer parameters, Reynolds number and Mach number, and the CF and CF0 mostly depends on the characteristics of the highest inflection point in the velocity profile. The three models were then combined in order to produce growth rates for the various kinds of boundary layer instabilities. These growth rates may be integrated to produce envelope \( N \)-factors exactly as in case of exact stability theory,

\[
N_{env} = \max_{f} \{ \max(\sigma_{TS}, \sigma_{CF0}, \sigma_{CF}) dx \}. 
\]

In practice, a method based on Fibonacci series is used to locate the wave direction producing the largest growth rate, after the definition of the range of variation of the \( \psi \) angle. This is done separately for \( N_{TS} \) and \( N_{CF} \). In case of the envelope, careful optimization has ensured that, in general, the most amplified frequencies are represented with a precision of about 1 count in \( N \)-factor, for \( N \approx 10 \), or about 10%. The transition \( N \)-factor must then be determined exactly like when using an exact stability code, with values at transition depending on the type of environment (flight or wind tunnel), and of the dominant type of instability, see ref. 18.

The two \( N \)-factors transition prediction method may also be used, with two options:

The first tries to resemble the classical \( N_{TS}/N_{CF} \) method, based on \( N_{CF0} \) and \( N_{TS} \) with \( \psi = 0 \) deg. (but uses compressible approach for compressible flows). The second takes into account a larger set of modes, as it includes traveling waves into \( N_{CF} \).
Concerning crossflow models, there remains a lack of precision for low frequencies, at the junction of the two models. Growth of low frequency traveling crossflow instabilities are underestimated, but zero frequency modes are correctly estimated. This does not seriously impact the N-factor curves and the quality of transition prediction.

The resulting code allows complete stability calculation with about two orders of magnitude reduction in computing time, compared to exact stability calculation. Another important advantage of the method is that it does not require any initial values, and is thus well adapted for insertion into boundary layer codes for fully automatic usage, like it has been done in the ONERA 3D boundary layer code 3C3D, but also in codes used by the European Research Centers INTA, FOI, CIRA and DLR, in the course of the European projects EUROLIFT, EUROLIFT 2 and SUPERTRAC. In case of 3C3D, this allows the stability computation over the complete surface of a vehicle, with a 3D mean flow (local stability hypothesis are still contained in the models).

Coupling to a RANS code is, on the other hand, more difficult because the incompressible shape factor must be evaluated with a good precision in the entire unstable region, and the crossflow criterion needs information about the upper part of the velocity profile, where usually the mesh density is rapidly reducing. Other difficulties arise with a RANS code, like the evolution of the transition location in the course of the calculation. Nevertheless, as exact stability computation can be realized on profiles extracted from a RANS field, there is no reason why this should not be accomplished.

**VIII. Applications**

First example corresponds to a low speed (80 m/s) swept (50 deg.) ONERA-D profile of 0.3 m chord, at -1 deg. incidence. The velocity distribution, on figure 11, shows a small peak on the upper side near the leading edge, followed by a negative pressure gradient. For the sake of clarity, only database results are shown on figure 11 for this case. Stationary and traveling crossflow instabilities are observed near the leading edge, in the negative pressure gradient region. Quite typical for this kind of flow, stationary instabilities dominate crossflow along 5 to 7 percent of chord, while traveling waves become larger further downstream. TS instabilities (φ=0 deg) start to grow at about 15 percent chord, where the negative pressure gradient becomes less pronounced, and rises more rapidly past the maximum velocity point. The four instability curves correspond to N_{TS}, N_{CF0}, N_{CF} and N_{ENV}. Concerning the envelope N-factor, crossflow contributes to its first part, roughly up to the maximum velocity point, and then TS waves are the dominant contribution.

All the curves are envelope over frequency, except for the stationary crossflow.

It should be kept in mind that this N_{CF0} is comparable to a zero frequency envelope N-factor, by definition with larger values that what would be obtained with LST based on constant β^Φ method (c.f. figure 4).

It is quite obvious from figure 11 that transition will correlate to different N-factors values depending on the definition. While a single N-factor is used with the envelope method, a curve has to be defined in case of two N-factor methods, transition being predicted when the point of coordinates (N_{CF}, N_{TS}) crosses it. See reference 23 for examples of application.

A second case, corresponding to a low speed “DTP-B” model at 40 deg. sweep is next considered (U∞ = 70 m/s, α = -4 deg.). Figure 12 shows the velocity distribution, and compares database N-factors with their exact LST counterparts. Again, this upper side at negative incidence shows an extended negative pressure gradient, without any leading edge peak, which promotes crossflow instabilities. TS waves start further downstream, resulting into a ‘mixed’ CF & TS case. Comparison to exact LST results shows differences below 2 points in N-factors, or about 10%, on the envelope curve, and even smaller for N_{TS} and N_{CF}.

A third example, presented on figure 13 with the same curves as figure 12, corresponds to a transonic case from the Fokker 100 Natural Laminar Flow flight experiments, which were run in the frame of the European project ELFIN. Boundary layers were computed using a conical wing hypothesis, while stability calculations proceed here assuming an infinite swept wing. The vertical arrow indicates where transition occurs, and the velocity distribution is also plotted on the figure. The database results are again compared to exact LST computations, with a fair agreement in this difficult test case. Departure from exact results remains lower than 2 at transition location, but with increasing difference as the database N_{CF0} remains at a constant value while the LST result continues growing. Five similar Fokker 100 cases were analyzed, bringing out the following correlations at transition in flight conditions:

- N_{TS} = 9, N_{CF} = 15, N_{CF0} = 11, N_{ENV} = 22 for TS dominated cases, and N_{ENV} = 15 for CF dominated cases.
In wind tunnel studies, with the proposed database method, typical values are $N_{CF} = 7$ and $N_{CF0} = 5$, but these values should be adapted for each test.

Application to transonic cryogenic tests (transonic small models at large Reynolds numbers) showed that the crossflow model is not, at present, well adapted because boundary layers become extremely thin, outside the validity domain of the model. On the other hand, the $N_{TS}/N_{CF0}$ approach remains valid in these configurations.

After application to a number of profiles, 2D and swept, the complete database method has been introduced as a transition prediction tool into the 3C3D boundary layer code, allowing the computation of amplification over the complete surface of a wing or a vehicle. In 3C3D, growth rates are calculated in the course of the parabolic boundary layer equations resolution, and then N-factors are integrated along external streamline directions, until transition (or a separation) is predicted. To illustrate the use of this tool, an application to the slat of a high lift configuration is presented on figure 14. The KH3Y configuration was extensively studied within the European projects EUROLIFT I and II. Results are presented here for the upper slat, for a wing incidence of 8.5 deg. and a chord Reynolds number of $4.15 \times 10^6$.

<table>
<thead>
<tr>
<th>Method</th>
<th>$N_{ENV}$ or $N_{TS}$</th>
<th>$N_{CF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Envelope</td>
<td>7.15</td>
<td>7</td>
</tr>
<tr>
<td>$N_{TS}/N_{CF}$</td>
<td>7.15</td>
<td>7</td>
</tr>
<tr>
<td>$N_{TS}/N_{CF0}$</td>
<td>7.15</td>
<td>5</td>
</tr>
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</table>

Table 1: Transition N-factor settings for the KH3Y slat

Figure 14: Application to the upper side of the KH3Y slat (8.5° incidence, $Re_c=4.15 \times 10^6$)
Three methods are used in this case, envelope, $N_{TS}/N_{CF}$, and $N_{TS}/N_{CF0}$. In the first case, the three models (for TS, CF and CF0) contribute to the envelope curve. In the second case, $N_{TS}$ is including waves with a $\psi$ direction such that $|\psi| < 60^\circ$, and $N_{CF}$ is based on traveling and stationary waves. In the last one, $N_{TS}$ is restricted to $\psi = 0^\circ$ and $N_{CF0}$ to $f = 0$. Figure 14 shows the various regions of the flow according to the three methods, but with the N-factors at transition given on table 1. The outer wing region is characterized by a large region (25% of span) where transition is caused by laminar separation. On each side an extended transition region is predicted, up to the trailing edge. Early transition is only predicted near the slat root with $N_{TS}/N_{CF0}$, resulting in a transition zone followed by a turbulent one. In each case, the unstable region is delimited into TS and CF regions, depending on the largest of the two N-factors. Comparison of the results seems to show the equivalence of the three methods, provided that proper settings are used. It would be most interesting to compare the spanwise evolution of transition with experimental results, but in most experiments transition measurements are usually done in a limited spanwise region.

A final example is presented on figure 15, which corresponds to a Mach 2 laminar wing designed and flight tested by JAXA. ONERA cooperated with JAXA on the pre and post-flight numerical transition analysis and performed wind-tunnel tests in the ONERA S2. Figure 15 shows, for a post flight case, a comparison of envelope N-factor curves obtained with two exact stability codes and with the database method. The agreement on the envelope curves is here again very satisfying. An extended version of the TS model was used for this case. $N_{TS}/N_{CF}$ curves are also shown, but with no comparison to exact calculation.

**IX. Conclusion**

Three separate models were developed for simplified prediction of TS, CF and stationary instabilities, aiming at allowing fast and automatic transition predictions. These three models rely heavily on Stuart theorem, stating in temporal theory the equivalence of the stability characteristics of a 3D velocity profile in direction $\phi$ with that of the projected 2D profile in the same direction, and on Gaster’s relation to transform the spatial growth rates. While integration should in principle be conducted along the group velocity direction, it is assumed that this direction remains very close to the external velocity direction. The same hypothesis is often used in exact stability codes. These models have been inserted inside the 3D boundary layer code 3C3D, and allow fully automatic transition predictions in a broad range of configurations. Extension to high lift cases, with strong acceleration, was realized with application to the KH3Y three elements wing. The two main advantages of these methods are that they do not require determination of initial values, and that the computing time is about two orders of magnitude smaller compared to exact, local stability.

These methods are presented as engineering tools; they do not intend to replace exact stability codes. They are most useful in design and when parametric variations are considered, like for optimization purposes. Emphasis has been placed here on 3D transonic configurations, but there exists an extension of the TS model to supersonic flows, up to Mach 4. This extension was used for the JAXA case presented in the paper. CIRA and ONERA also used the database approach in the design of the Mach 2 SUPERTRAC wing, in which case it was introduced into an optimization process.

In case of a boundary layer, computation time is usually very small, and in 3C3D the three models and N-factor curves are in fact computed, allowing comparison of the results. For the integration into a RANS code, selection of a single method would be necessary in order to reduce computing time to a minimum, because of the iterative process. Two methods are most cost effective, the envelope method and the $N_{TS}/N_{CF0}$. Introduction into a RANS code has not yet been realized, although these methods can be used with velocity profiles extracted from RANS solution fields, like exact stability computation. In such case, care must be taken to select a low dissipation numerical scheme, like the Roe scheme, and to ensure a minimum of about 40 mesh points in the viscous region.

ONERA database methods were developed for transonic aeronautic applications, with great emphasis on crossflow instabilities, and are not applicable to separated boundary layers. Extension to these cases should be easily attained, if needed, as the first study identified the relevant parameters which should be used.
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