A NEW STABILITY APPROACH FOR THE FLOW INDUCED BY WALL INJECTION

G. Casalis,[1] F. Chedevergne,[1] Th. Feraille,[1] and G. Avalon[2]

[1] *ONERA/DMAE, 2, avenue Ed. Belin, 31 055 Toulouse Cedex, FRANCE*
gregoire.casalis@onecert.fr, francois.chedevergne@onecert.fr

[2] *ONERA/DEFA, chemin de la Huniere, 91 761 Palaiseau Cedex, FRANCE*
gerard.avalon@onera.fr

Abstract The present paper deals with the stability analysis of the flow induced by wall injection either in a rectangular duct or in a cylindrical pipe. As the basic flow is strongly nonparallel, the modal form gives amplitude functions which are dependent on two space variables. The linear stability problem is thus described by a PDE system whose results seem to be in good agreement with the experiments.

Introduction

Under some conditions, large solid rocket motors may exhibit thrust oscillations. In order to analyse this phenomenon, cold gas experiments are carried out. As suggested by numerical simulations, e.g. Lupoglazoff and Vuillot, 1996, the velocity field in the cold gas set-up is expected to reproduce faithfully the one occurring in real motors with combustion.

In the framework of the instabilities modelling, the main difficulty in the present case deals with the basic flow which is strongly non parallel. Usual stability approaches are performed within the parallel assumption. In this case, using the normal mode form, the amplitude functions depend on one space variable only and the linearized Navier-Stokes equations lead to an ordinary differential equation of Orr-Sommerfeld type. This has been successfully applied to shear flows such as the boundary layer or the jet flows. However there are some physical configurations for which the basic flow is fully non parallel, its velocity field depends on two space variables (instead of only one), like a separated boundary layer and the boundary layer around an attachment line. In these cases, the normal mode assumption leads to amplitude functions which depend on two variables and the stability equations then become an eigenvalue problem written as a system of partial differential equations.

This second approach needs obviously large computational resources mainly concerning the memory, but the major difficulty seems to be related to the boundary conditions, except for the case of the Poiseuille flow, see Tatsumi and Yoshimura, 1990. With non physical boundaries, the published results are more recent, see Lin and Malik, 1996, Theofilis, 1998 and Robinet and de la Motte, 2003 for instance. The present paper gives an outline for a flow which presents only one non physical boundary, details can be found in Féraille, 2004. The obtained results are given in comparison with measurements obtained with two cold gas set-ups, one is a planar duct, the other one a cylindrical pipe.

1. Physical model

Theoretical model

Two configurations are analyzed, case 1 is a rectangular duct, case 2 a circular pipe, see figures below. In both cases, air is uniformly in-

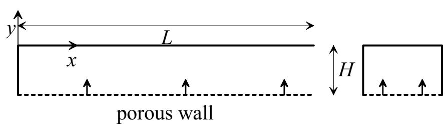

Figure 1. Scheme of VECLA set-up.

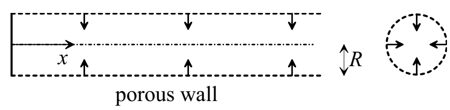

Figure 2. Scheme of VALDO set-up.

jected through a porous wall. All quantities given below are made dimensionless thanks to the height H or R and the norm of the injection velocity. The characteristic Reynolds number based on these quantities is noted $\mathcal{R}_e$. In some conditions the flow is laminar at least for small values of x. A laminar analytical steady inviscid solution exists ; in both cases, corresponding streamfunctions are expressed by :

$$\Psi_1 = x \sin\left(\frac{\pi y}{2}\right) \qquad \Psi_2 = x \sin\left(\frac{\pi r^2}{2}\right) \tag{1}$$

where subscripts 1 and 2 correspond respectively to the rectangular duct and the circular pipe. For the considered large Reynolds numbers, this form is accurate enough, see Casalis et al., 1998.

The flow is strongly non parallel, particularly for small values of x, see Figure 3. A linear stability analysis of this flow is carried out. Assuming that the perturbation remains two-dimensional (case 1) or axisymmetric (case 2), a streamfunction ϕ may be associated to the fluctuation. Due to the non parallel nature of the basic flow, the normal mode writes as :

$$\phi_1(x, y, t) = \hat{\phi}_1(x, y)e^{-i\omega t} \qquad \phi_2(x, r, t) = \hat{\phi}_2(x, r)e^{-i\omega t}$$

with ω a complex number, its real part ω_r corresponds to the circular frequency of the instability mode and its imaginary part ω_i to the temporal growth rate. The amplitude function thus depends on two space variables and the linear stability problem consists in solving a partial differential equation (PDE) for ϕ_1 (resp. ϕ_2) with respect to (x, y) (resp. (x, r)) and ω is the eigenvalue to be determined.

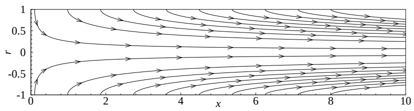

Figure 3. Steady streamlines, case 2.

The PDE has to be solved in a rectangular domain. The transverse coordinate y (or r) varies between the symmetry line and the porous wall and the axial coordinate x varies from the front wall $x = 0$ up to a given exit value X_e. On the first boundary, symmetry conditions are imposed, on the second and third ones the fluctuating velocity is imposed to be zero, the fourth boundary is artificial and "ad-hoc" conditions are imposed (see section 2).

Experimental set-ups

The two configurations of figures 1 and 2 have been experimentally explored with the set-ups VECLA and VALDO using the same feeding equipment. Air coming from a pressurized tank at 250 bar is injected inside the two set-ups through elementary throats that control the mass flow rate entering the duct at several parts of the porous wall. To compensate the decrease of temperature due to its depressurization, air passes inside a gas heater before its injection. The heat quantity given to the air is controlled by a thermo-regulator whose power is adjusted to obtain a temperature of 20°C inside the ducts.

The porous walls of the two set-ups are formed of bronze poral obtained by joining together small spheres of bronze of same diameter. Low porosities, typically equal to 8 μm or 18 μm, are adopted for these porous walls in order to avoid as much as possible the transfer of acoustic energy from the duct to the backside of the porous wall. The VECLA set-up is composed of a planar chamber which is 603 mm in length and 60 mm in width whose bottom is equipped with a porous plate of 5 mm thickness and 581 mm in length. The height of the duct can be fixed to 10 mm, 20 mm, 30 mm or 40 mm by mounting metallic blocks under the top wall of the chamber which contains special ports where pressure transducers and hot wire can be introduced. A nozzle can be attached to the downstream end of the chamber. The VALDO set-up is of modular type with a conception in separate modules, each of them containing a porous cylinder of 60 mm in diameter, made in porous bronze of thickness 5 mm. Air is injected into each module by three orifices which are regularly spaced around the circumference. The modules have a length of 168 mm and, since four modules are available at the present time, the maximum channel length is 672 mm. Over the four modules, one has several ports located in front of holes drilled on the surface of the porous cylinders at which the pressure, the temperature and the velocity of the air inside the central duct can be measured. The radial displacement of the hot wire is ensured by a pilotable table whose variation extent is of 100 mm. The set-up can operate in two versions depending if a nozzle attached to the end of the injecting part is used or not.

2. Numerical procedure

The PDE system written for $\hat{\phi}$ (case 1 or 2) is discretized by a spectral collocation method in the two space directions. The problem becomes a generalized eigenvalue problem $\underline{\underline{A}}.\underline{X} = \omega \underline{\underline{B}}.\underline{X}$ with $\underline{\underline{A}}$ and $\underline{\underline{B}}$ two matrices and $\underline{X}$ the vector corresponding to the value of $\hat{\phi}$ on each double collocation point. Due to the size of the matrices, the spectrum (set of the eigenvalues ω) is obtained part by part using an Arnoldi algorithm, see Arnoldi, 1951. This means that a target is specified before each calculation and only the eigenvalues close to it are computed. The obtained results consist in a set of discrete complex values for ω. With 100 collocation points in x and 120 in r (case 2), table 1 gives the numerical

Table 1. Converged numerical eigenvalues for five modes, $\mathcal{R}_e = 1000$, case 2

k	1	2	3	4	5
ω_r	5.4776	10.189	14.378	18.130	21.536
ω_i	-4.9307	-6.8131	-8.5493	-10.258	-11.810

values of some modes identified by the integer k, see figure 5.

After several attempts, a simple extrapolation for $\hat{\phi}$ is imposed at the boundary X_e. The results (eigenvalue and eigenfunction) are found amazingly to be independent of the type of conditions imposed at $x = X_e$ (other conditions have been tested, see Féraille, 2004) and are also independent of the location of the exit abscissa X_e. This is clearly shown in figure 4, which gives the contours of $|Re(\hat{\phi}_2)|$ for four values of X_e. Except maybe in a region very close to X_e, each result is completely superposed to the other results obtained for larger values of X_e. This means that the general structure of the mode is determined by the upstream part of the flow : moving the non physical X_e downstream does not affect the upstream physical values associated to the eigenmode.

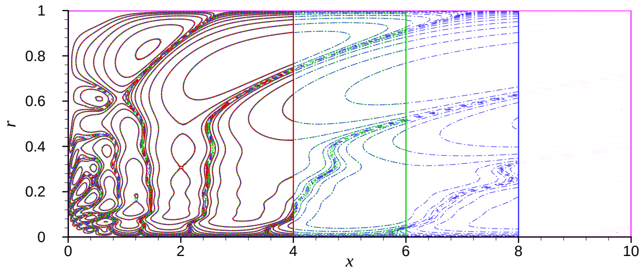

Figure 4. Norm of the real part of $\hat{\phi}_2$ associated to the mode $k = 5$. Superposed results obtained with four different exit sections : 4, 6, 8 and 10, $\mathcal{R}_e = 2100$.

3. Stability results

The first result is the spectrum i.e. the set of the complex eigenvalues ω. For both configurations the spectrum is plotted in figure 5 in the complex plane (ω_r, ω_i). Several observations may be done, they are

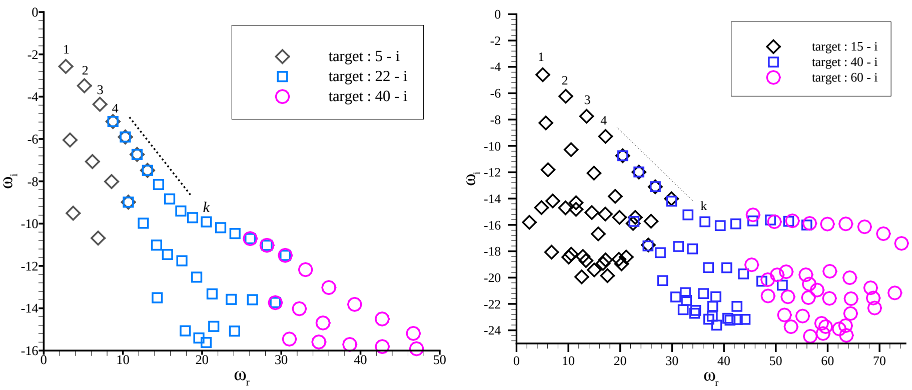

Figure 5. Spectrum in the complex (ω_r, ω_i) plane : case 1 (left) with $\mathcal{R}_e = 1000$ and case 2 (right) with $\mathcal{R}_e = 2100$

the same in both cases. Only (temporally) damped modes are obtained seeing that the temporal growth rates are all negative. The basic flow is thus stable from this point of view. The eigenfunctions especially for frequencies (dimensionless values) between 20 and 80 exhibit actually a huge growth in the x direction, in fact a growth which is nearly exponential. The modes are stable with respect to the time but exponentially growing in space (with respect to x) ! It can be also remarked that only discrete values of the frequency are obtained. This result contradicts the conclusion obtained by using the classical normal mode approach (assuming that the basic flow is parallel whereas it is clearly not, see figure 3). A continuous range of frequencies corresponding to spatially amplified modes is predicted by this usual stability analysis, see Casalis et al., 1998. Comparison with the experiments is given in figure 6. The

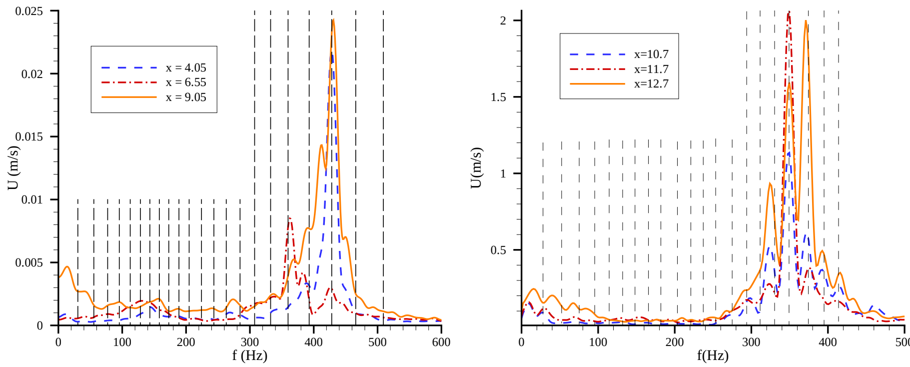

Figure 6. Comparisons between the new stability analysis and measurements of the fluctuating velocity by a hot wire : case 1 (left) and case 2 (right)

vertical dashed lines correspond to the dimensional values of the different discrete modes k shown in figure 5. A rather good agreement is obtained especially in case 2 and the experimental results seem to be closer to a discrete structure than exhibiting a continuous range of amplified frequencies. The modes are temporally damped, are exponentially growing in x and only some of them are measured. This may indicate that the environmental fluctuations are very important in terms of receptivity.

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